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AN OPTIMAL PROBLEM OF SATELLITE GUIDANCE BY MEANS OF A GYROSCOPE

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The use of gyroscopes for attitude cantrol and stabilization of space vehicles in the case of large angles is considered.

The simplest formulation of this nonlinear problem is investigated. An artificial earth satellite is equipped with a balanced two-axis gyro in a gimbal mount which acts as its final control element. The center of inertia of the gyro coincides with the center of inertia of the satellite body, and the axis of the outer gimbal (output axis) is parallel to one of the principal axes of inertia of the vehicle. It is assumed that the system is not acted on by external moments, so that its moment of momentum vector remains constant.

After stabilization of the angular position of the satellite on its orbit, i. e. after elimination of the initial angular velocities of the system, the entire moment of momentum is borne by the gyro wheel. The system can be rotated by altering the position of the gyro wheel axis (spin axis); the controls are the moments M_α and M_β acting on the gimbal axes. The angles of rotation α and β of the gimbals are called the "control angles".

Although the results obtained are largely qualitative in character, they can be used in conjunction with the iteration method to construct a more exact solution.

One of the two controls in the control mode just described, namely β, is varied relay fashion. The angle α, i. e. the rotation of the outer gimbal between the initial and final rapid rotations, is varied periodically and depends on the angle of nutation ϑ and on the inertial characteristic of the system.

During guidance the z-axis describes looped ($n < 0$) or wavy ($n > 0$) curves on a fixed unit sphere; these curves are bounded by two parallels for which $\sin\vartheta = \pm n$. The self-intersection points of the loops or the inflection points of the wavy curves correspond to $\vartheta = \vartheta_0$.

Let the initial position of the satellite body be known and let the purpose of control be to achieve a certain attitude change, i. e. let the final position of the vehicle in space be specified. As the spin axis rotates in the satellite body and in inertial space, the satellite body acquires an angular velocity in accordance with the law of conservation of the moment of momentum. The law of variation of the controls M_α and M_β and the rotation time must be chosen in such a way that the system experiences the required attitude change. The problem is obviously not single-valued. The optimal problem is formulated with the rotation time as the optimality criterion. There are no restrictions either on the controlling moments M_α and M_β or on the guidance time, which in this case likewise plays the role of a control and is not independent of the required rotation.

1. The equations of motion. We introduce two coordinate systems (Fig. 1). The right-handed system $Oxyz$ is attached to the body, its origin lies at the center of mass O; the x-, y-, and z-axes coincide with the principal central axes of inertia of the body.

The system $O\xi\eta\zeta$ is a right-handed coordinate system with its origin at the same point O; the ξ-, η-, and ζ-axes retain their direction in inertial space. The position of the attached coordinate system relative to the inertial system is characterized by the three Euler angles ϑ, φ, ψ. The initial position of the body is defined by the angles ϑ_0, φ_0, ψ_0 and its final position by the angles ϑ_1, φ_1, ψ_1. Without limiting generality we can assume that the axis $O\zeta$ coincides with the initial position of the spin axis when both gimbals and the body are in a stationary position. In terms of the chosen coordinate systems this means that the line of nodes On coincides with the axis of the casing (Fig. 1).

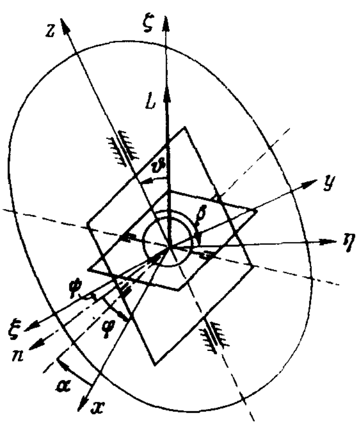

Fig. 1

Let $\mathbf{L}$ denote the kinetic moment of the system relative to the point O, let D be the tensor of inertia of the wheel (for simplicity we shall assume that it is spherical, denoting the moment of inertia relative to any central axis by D), and let ω_0 be the absolute angular velocity of the wheel when the body is in a stationary position. These quantities are related by the expression

$$\mathbf{L} = \mathbf{D}\cdot\boldsymbol{\omega}_0 \tag{1.1}$$

Rotation of the gimbals is accompanied by rotation of the body, and the moment of momentum $\mathbf{L}$ remains constant; its absolute value $L = D\omega_0$, and $\mathbf{L}$ coincides with ζ. Denoting the tensor of inertia of the body in the coordinate system xyz by $\mathbf{J}$, the angular velocity of the body by ω, and the absolute angular velocity of the wheel for a moving body by ω_1 (neglecting the gimbal masses), we obtain

$$D\boldsymbol{\omega}_0 = \mathbf{J}\boldsymbol{\omega} + \mathbf{D}\boldsymbol{\omega}_1 \tag{1.2}$$

If A, B, C are the principal central moments of inertia of the system, then after several operations we can express (1.2) as

$$\begin{aligned}
&A\vartheta^{\cdot}\cos\varphi + A\psi^{\cdot}\sin\vartheta\sin\varphi - D\beta^{\cdot}\cos\alpha + D\omega_0\sin\beta\sin\alpha = \\
&\qquad = D\omega_0\sin\vartheta\sin\varphi \\
&-B\vartheta^{\cdot}\sin\varphi + B\psi^{\cdot}\sin\vartheta\cos\varphi + D\beta^{\cdot}\sin\alpha + D\omega_0\sin\beta\cos\alpha = \\
&\qquad = D\omega_0\sin\vartheta\cos\varphi \\
&C\varphi^{\cdot} + C\psi^{\cdot}\cos\vartheta - D\alpha^{\cdot} + D\omega_0\cos\beta = D\omega_0\cos\vartheta
\end{aligned} \tag{1.3}$$

In order to obtain the closed set of equations of motion we must also write out the equations for the variations of the angles α and β under the action of the controlling moments M_α and M_β. The theorem on the variation of the moment of momentum applied to the gyroscope yields

$$\Gamma_0^{\cdot} + \omega_* \times \Gamma_0 = \mathbf{M}_\alpha + \mathbf{M}_\beta \tag{1.4}$$

where Γ_0 is the absolute moment of momentum of the gyro wheel relative to the point O, and ω_* is the angular velocity vector of the coordinate system in which the derivative $\Gamma_0^{\cdot}$ is taken. In terms of projections on the outer and inner gimbal axes, Eq. (1.4) yields the relations

$$\begin{gathered}
M_\alpha = -D\alpha^{\cdot\cdot} + D\varphi^{\cdot\cdot} + D\psi^{\cdot\cdot} - D\vartheta^{\cdot}\psi^{\cdot}\sin\vartheta - D\beta^{\cdot}\vartheta^{\cdot}\sin(\varphi-\alpha) + \\
+ D\beta^{\cdot}\psi^{\cdot}\sin\vartheta\cos(\varphi-\alpha) + D\omega_0\vartheta^{\cdot}\sin\beta\cos(\varphi-\alpha) + \\
+ D\omega_0\psi^{\cdot}\sin\beta\sin\vartheta\sin(\varphi-\alpha) - D\omega_0\beta^{\cdot}\sin\beta
\end{gathered}$$

$$\begin{gathered}
M_\beta = D\beta^{\cdot\cdot} - D\vartheta^{\cdot\cdot}\cos(\varphi-\alpha) - D\psi^{\cdot\cdot}\sin\vartheta\sin(\varphi-\alpha) + \\
+ D\vartheta^{\cdot}(\varphi^{\cdot}-\alpha^{\cdot})\sin(\varphi-\alpha) - D\psi^{\cdot}(\varphi^{\cdot}-\alpha^{\cdot})\sin\vartheta\cos(\varphi-\alpha) - \\
- D\vartheta^{\cdot}\psi^{\cdot}\cos\vartheta\sin(\varphi-\alpha) + D\omega_0\vartheta^{\cdot}\cos\beta\sin(\varphi-\alpha) - \\
- D\omega_0\psi^{\cdot}\cos\beta\sin\vartheta\cos(\varphi-\alpha) + D\omega_0\psi^{\cdot}\sin\beta\cos\vartheta + D\omega_0(\varphi^{\cdot}-\alpha^{\cdot})\sin\beta
\end{gathered} \tag{1.5}$$

Equations (1.3) and (1.5) form the closed set of equations of motion of the system and gyro wheel in the case where the expressions for the controlling moments M_α and M_β are specified. Since we shall be investigating the optimal problem, it is expedient at this point to make certain simplifications in order to lower the order of this set of equations. Linearizing Eqs. (1.3) under the assumption that the variations of the angles α, β and of the Euler angles are small, as is in fact the case over a very small time interval Δt, we obtain the following set of equations:

$$\begin{gathered}
\Delta\vartheta = D\omega_0\left(\frac{1}{A}-\frac{1}{B}\right)\sin\vartheta\sin\varphi\cos\varphi\,\Delta t + \\
+ D\left(\frac{1}{A}\cos\alpha\cos\varphi + \frac{1}{B}\sin\alpha\sin\varphi\right)\Delta\beta - \\
- D\omega_0\sin\beta\left(\frac{1}{A}\sin\alpha\cos\varphi - \frac{1}{B}\cos\alpha\sin\varphi\right)\Delta t \\
\Delta\varphi = \frac{D}{C}\omega_0\cos\vartheta\Delta t - \frac{D}{C}\omega_0\cos\beta\Delta t + \frac{D}{C}\Delta\alpha - \Delta\psi\cos\vartheta \\
\Delta\psi = \frac{1}{\sin\vartheta}\left[D\Delta\beta\left(\frac{1}{A}\cos\alpha\sin\varphi - \frac{1}{B}\sin\alpha\cos\varphi\right) - \right. \\
- D\omega_0\sin\beta\left(\frac{1}{A}\sin\alpha\sin\varphi + \frac{1}{B}\cos\alpha\cos\varphi\right)\Delta t + \\
\left. + D\omega_0\sin\vartheta\left(\frac{1}{A}\sin^2\varphi + \frac{1}{B}\cos^2\varphi\right)\Delta t\right]
\end{gathered} \tag{1.6}$$

Here $\Delta\vartheta$, $\Delta\varphi$, $\Delta\psi$ are the increments of the Euler angles and $\Delta\alpha$, $\Delta\beta$ are the control increments occurring with rapid rotation of the gimbals over a very short time interval Δt such that $\omega_0\Delta t$ is commensurate with $\Delta\alpha$ and $\Delta\beta$.

We know from the literature that the ratios D/A, D/B, D/C are small quantities on the order of 1/100. If the gimbals are rotating slowly, i.e. if $\alpha^{\cdot}$ and $\beta^{\cdot}$ are small compared with ω_0, the terms containing the expressions $D\alpha^{\cdot}$ and $D\beta^{\cdot}$ can be neglected in the first approximation in comparison with the other terms in (1.3). Equations (1.3) can then be solved independently of (1.5) by considering the kinematic quantities β and α as the controls instead of the true controls, i.e. of the moments M_α and M. Essentially, the problem reduces to the determination of the controls α and β from the conditions imposed on the required final rotation and on the minimization of the rotation time. It is then possible to determine the controlling moments from Eqs. (1.5) and to proceed with more exact solution of the problem.

According to Eqs. (1.6), during rapid rotation of the gimbals large changes in the angles α and β (i.e. $\alpha^\cdot$ and $\beta^\cdot$ commensurate with ω_0) correspond to insignificant changes of the Euler angles which become smaller as the rate of rotation of the spin axis increases. An exception to this is the highly specific case where $\vartheta = 0$. A more exact quantitative investigation of this process following solution of the problem under consideration yields corrections in terms of small angles.

This formulation enables us to neglect the changes in the Euler angles associated with rapid rotations of the spin axis; in the case of slow rotation of the spin axis the equations of motion of the supporting body assume the simplified form

$$\vartheta^\cdot = D\omega_0\left(\frac{1}{A} - \frac{1}{B}\right)\sin\vartheta\sin\varphi\cos\varphi + D\omega_0\sin\beta\left(-\frac{1}{A}\sin\alpha\cos\varphi + \frac{1}{B}\cos\alpha\sin\varphi\right)$$

$$\varphi^\cdot = D\omega_0\left(\frac{1}{C} - \frac{1}{A}\sin^2\varphi - \frac{1}{B}\cos^2\varphi\right)\cos\vartheta - D\omega_0\frac{1}{C}\cos\beta + D\omega_0\sin\beta\operatorname{ctg}\vartheta\left(\frac{1}{A}\sin\alpha\sin\varphi + \frac{1}{B}\cos\alpha\cos\varphi\right)$$

$$\psi^\cdot = D\omega_0\left(\frac{1}{A}\sin^2\varphi + \frac{1}{B}\cos^2\varphi\right) - D\omega_0\frac{\sin\beta}{\sin\vartheta}\left(\frac{1}{A}\sin\alpha\sin\varphi - \frac{1}{B}\cos\alpha\cos\varphi\right) \quad (1.7)$$

2. Solution of the optimal problem. Equations (1.7) become simpler if the moments of inertia A and B are equal. In this case we have

$$\vartheta^\cdot = \sin\beta\sin(\varphi - \alpha)$$

$$\varphi^\cdot = (\varepsilon - 1)\cos\vartheta - \varepsilon\cos\beta + \sin\beta\operatorname{ctg}\vartheta\cos(\varphi - \alpha) \qquad (2.1)$$

$$\psi^\cdot = 1 - \sin\beta\cos(\varphi - \alpha) / \sin\vartheta$$

The above expressions were obtained by setting $\tau = It / A$; however, we have left the symbols for the derivatives unchanged, even though $\vartheta^\cdot$, $\varphi^\cdot$ and $\psi^\cdot$ are now derivatives with respect to τ. The quantity $\varepsilon = A / C$ is called the "inertial characteristic". We note that this case (i.e. the case where the inertial ellipsoid is an ellipsoid of revolution) obtains in most of the satellite designs described in the literature.

We now pose the optimal problem with the aid of the maximum principle. The Hamiltonian of the problem can be written as

$$H = \vartheta^\cdot p_\vartheta + \varphi^\cdot p_\varphi + \psi^\cdot p_\psi \qquad (p_\vartheta^\cdot = -\partial H/\partial\vartheta,\ p_\varphi^\cdot = -\partial H/\partial\varphi,\ p_\psi^\cdot = -\partial H/\partial\psi)$$

The optimality conditions are

$$\partial H / \partial\alpha = 0, \qquad \partial H / \partial\beta = 0$$

The cyclical character of the coordinate ψ implies that

$$p_\psi^\cdot = 0, \qquad p_\psi = \text{const} = \mu$$

and the fact that the quantities φ and α occur together only (i.e. only in the combination $\varphi - \alpha$) in (2.1) imply that

$$p_\varphi^{\bullet} = -\partial H/\partial\varphi = \partial H/\partial\alpha = 0 \quad \text{or} \quad p_\varphi^{\bullet} = 0, \quad p_\varphi = \text{const} = \lambda$$

The optimal problem becomes considerably simpler if we do not impose restrictions on the finite characteristic rotation angle φ and set $\lambda \equiv 0$. Geomertically this means ensuring that the required orientation of the principal axis of inertia z of the system (which is the axis of symmetry of its ellipsoid of inertia and the gyro output axis) is achieved in the shortest possible time. In other words, we are required to solve the problem of orienting the plane xy in the minimal time.

The Hamiltonian in this case can be written as

$$H = \vartheta^{\bullet} p_\vartheta + \psi^{\bullet} p_\psi$$

or

$$H = \sin\beta \sin(\varphi - \alpha)\, p_\vartheta + \left[1 - \sin\beta \frac{\cos(\varphi - \alpha)}{\sin\vartheta}\right]\mu \qquad (2.2)$$

Optimization with respect to the angle α on the basis of the condition $\partial H / \partial\alpha = 0$ yields

$$\operatorname{tg}(\varphi - \alpha) = -1 / \mu\, p_\vartheta \sin\vartheta$$

The condition $\partial H / \partial\beta = 0$ implies that

$$\cos\beta \cos(\varphi - \alpha)\,[p_\vartheta^2 \sin^2\vartheta + \mu] = 0$$

Hence, $\cos\beta = 0$. Let us assume that $\sin\beta > 0$, so that $\beta = {}^1/_2\pi$. To find p_ϑ we can make use of the integral $H = 1$.

Eliminating the control α form H and setting $\beta = {}^1/_2\pi$, we obtain

$$p_\vartheta = \pm \frac{1}{\sin\vartheta} \sqrt{(1-\mu)^2 \sin^2\vartheta - \mu^2} \qquad (2.3)$$

Here it is convenient to set

$$n^2 = \mu^2 / (1-\mu)^2, \qquad n = \mu / |1 - \mu|$$

In this notation the angle $\varphi - \alpha$ is defined by the expressions

$$\cos(\varphi - \alpha) = -\frac{n}{\sin\vartheta}, \qquad \sin(\varphi - \alpha) = \pm\frac{1}{\sin\vartheta}\sqrt{\sin^2\vartheta - n^2} \qquad (2.4)$$

The signs in front of p_ϑ and $\sin(\varphi - \alpha)$ must be determined from the condition of maximum H. If $\vartheta^{\bullet}$ is positive, the upper signs in front of p_ϑ and $\sin(\varphi - \alpha)$ apply; the lower signs apply for a negative $\vartheta^{\bullet}$.

The differential equations of motion of the system become

$$\vartheta^{\bullet} = \pm\frac{1}{\sin\vartheta}\sqrt{\sin^2\vartheta - n^2}, \qquad \varphi^{\bullet} = (\varepsilon - 1)\cos\vartheta - \frac{1}{\sin^2\vartheta}\, n\cos\vartheta$$

$$\psi^{\bullet} = 1 + \frac{n}{\sin^2\vartheta} \qquad (2.5)$$

The domain of definition of the problem is specified by the condition $\sin^2\vartheta - n^2 \geqslant 0$. Since $|\sin\vartheta| \leqslant 1$, it follows that the coefficient n can assume the values $-1 \leqslant n \leqslant 1$, and the Lagrange multiplier μ varies from $+0.5$ to $-\infty$. Each specific n ($|n| \leqslant 1$) corresponds to a specific motion of the system; variation of the angle ϑ is restricted by the condition $|\sin\vartheta| \leqslant n$. Since $\sin\vartheta$ is a positive quantity, we can assume that the angle ϑ is restricted by the condition

$$\arcsin n \leqslant \vartheta \leqslant \pi - \arcsin n \qquad (2.6)$$

Equations (2.5) are integrable in quadratures. For $\tau_0 = 0$ we obtain

$$\cos\vartheta = \mp\sqrt{1 - n^2}\sin(\tau + \delta) \qquad \sin\delta = \mp\frac{\cos\vartheta_0}{1 - n^2} \qquad (2.7)$$

The upper signs must be taken for a positive $\dot{\vartheta}$. Figure 2 shows ϑ as a function of τ for $\vartheta_0 = {}^1/_2\pi$ for values of n from −1 to +1 in steps of 0.1.

The phase trajectories ϑ, ψ are defined by the differential equation

$$\frac{d\psi}{d\vartheta} = \pm \frac{\sin\vartheta}{\sqrt{\sin^2\vartheta - n^2}} \pm \frac{n}{\sin\vartheta\sqrt{\sin^2\vartheta - n^2}} \tag{2.8}$$

The integral of Eq. (2.8) is of the form

$$n \operatorname{ctg}\vartheta = \mp \sin(\Delta\psi + \gamma)\sqrt{\sin^2\vartheta - n^2} - \cos(\Delta\psi + \gamma)\cos\vartheta \tag{2.9}$$

$$(1 - n)\sin\gamma = \mp \operatorname{ctg}\vartheta_0 \sqrt{\sin^2\vartheta_0 - n^2}, \qquad \Delta\psi = \psi - \psi_0$$

The character of the phase trajectories does not depend on the inertial characteristic ε. Figure 3 shows the phase trajectories on the plane ϑ, ψ for $\vartheta_0 = {}^1/_2\pi$.

The differential equation of the phase trajectory ϑ, φ is

$$\frac{d\varphi}{d\vartheta} = \pm(\varepsilon - 1)\frac{\sin\vartheta\cos\vartheta}{\sqrt{\sin^2\vartheta - n^2}} \mp \frac{n\cos\vartheta}{\sin\vartheta\sqrt{\sin^2\vartheta - n^2}} \tag{2.10}$$

Its integral is

$$\varphi - \varphi_0 = \pm(\varepsilon - 1)\sqrt{\sin^2\vartheta - n^2} \mp \arccos\frac{n}{\sin\vartheta} \mp$$

$$\mp(\varepsilon - 1)\sqrt{\sin^2\vartheta_0 - n^2} \pm \arccos\frac{n}{\sin\vartheta_0} \tag{2.11}$$

Equations (2.4), (2.5) and (2.10) enable us to find the control α as a function of ϑ,

$$\alpha - \alpha_0 = \pm\sqrt{\sin^2\vartheta - n^2}(\varepsilon - 1) \pm \sqrt{\sin^2\vartheta_0 - n^2}(1 - \varepsilon) \quad \left(\cos\alpha_0 = -\frac{n}{\sin\vartheta_0}\right) \tag{2.12}$$

The parameter n must be chosen on the basis of Eq. (2.9). For specified ϑ_0, ψ_0 this equation yields a one-parameter family of curves from which we must select the curve passing through the required point with the coordinates ϑ_1, ψ_1. Analytically this condition is expressed by the equation

$$\sin\Delta\psi\sqrt{\sin^2\vartheta_1 - n^2}\sqrt{\sin^2\vartheta_0 - n^2} \pm \cos\Delta\psi\cos\vartheta_1\sqrt{\sin^2\vartheta_0 - n^2} \mp$$

$$\mp \cos\Delta\psi\cos\vartheta_0\sqrt{\sin^2\vartheta_1 - n^2} + \sin\Delta\psi\cos\vartheta_1\cos\vartheta_0 =$$

$$= \mp n\frac{\operatorname{ctg}\vartheta_1}{\sin\vartheta_0}\sqrt{\sin^2\vartheta_0 - n^2} \pm$$

$$\pm n\frac{\operatorname{ctg}\vartheta_0}{\sin\vartheta_1}\sqrt{\sin^2\vartheta_1 - n^2} \tag{2.13}$$

from which we can compute n. The upper signs in (2.8)–(2.13) must be taken for a positive $\dot{\vartheta}$.

It is sometimes more convenient to select the parameter n graphically. The phase trajectories for different permissible values of n ($|n| \leqslant \sin\vartheta_0$) and for ϑ_0 of differing sign must be constructed on the phase plane ϑ, ψ for each initial value $\dot{\vartheta}$. The final point with the coordinates $\vartheta_1, \Delta\psi = \psi_1 - \psi_0$ can be found on the same plane; the value of n for the trajectory passing

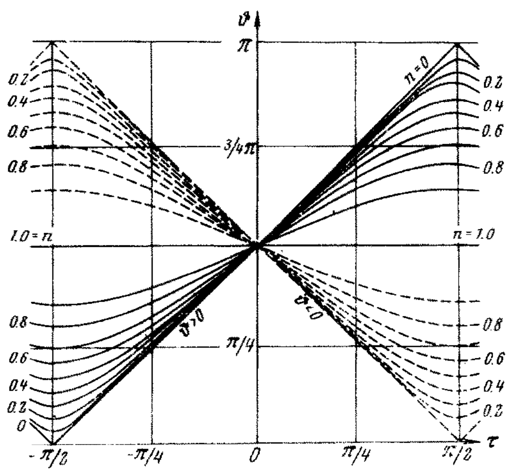

Fig. 2

through it can be calculated by interpolation.

The problem always has a solution. The only special case is $\vartheta_0 = 0$ or $\vartheta_0 = \pi$. We note that the domain of possible parameters n contracts as the initial point moves away from the straight line $\vartheta_0 = {}^1/_2\pi$. The n thus chosen must be substituted into Eq. (2.12) in order to obtain the corresponding mode of variation of the control α. The same n determines the equations of motion of the system. The rotation time τ_1 can be determined from (2.7) by setting $\vartheta = \vartheta_1$.

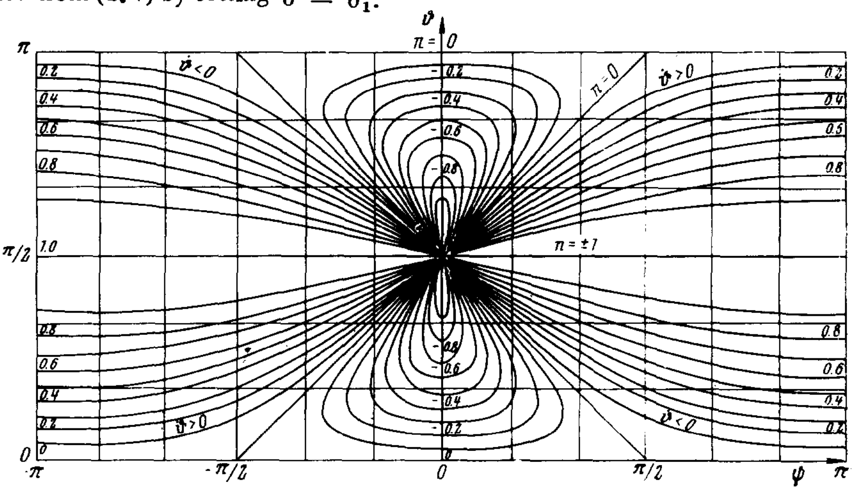

Fig. 3

3. Variation of the control angles. In the initial position (when the supporting body is in a stationary position) the angles α and β can be denoted by $\alpha_0{}^*$ and $\beta_0{}^*$. They must satisfy the static relations (Eqs. (2.1)) under the assumption that $\dot\vartheta = \dot\varphi = \dot\psi = 0$,

$$\alpha_0{}^* = \varphi_0, \qquad \beta_0{}^* = \vartheta_0 \tag{3.1}$$

In the case of optimal control, the corresponding control functions at the start of motion of the gimbals, when $\vartheta = \vartheta_0$, $\varphi = \varphi_0 = 0$, are

$$\beta_0 = {}^1/_2\,\pi, \qquad \cos\alpha_0 = -n / \sin\vartheta_0 \tag{3.2}$$

They clearly differ from $\beta_0{}^* = \vartheta_0$ and $\alpha_0{}^* = \varphi_1$, so that the spin axis must be reorientated prior to the start of rotation from the position defined by the angles $\alpha_0{}^*, \beta_0{}^*$ to the position defined by α_0, β_0 at the maximum permissible a_{max} and β_{max}. As soon as the body has attained the required angles ϑ_1, ψ_1 the wheel must be returned to its initial position in inertial space at the maximum permissible $\dot\alpha_{max}$ and $\dot\beta_{max}$. Thus, the wheel again bears the entire moment of momentum, and the system stops rotating. The angles

$$\alpha_1{}^* = \varphi_1, \ \beta_1{}^* = \vartheta_1 \tag{3.3}$$

are then clearly different from $\alpha_0{}^*$ and $\beta_0{}^*$.

The maximum permissible α_{max} and $\dot\beta_{max}$ must be determined from Eqs. (1.5) on the basis of the maximum permissible moments $M_{\alpha max}$ and $M_{\beta max}$.

Translated by A.Y.